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Spectral synthesis on zero-dimensional locally compact Abelian groups

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Спектральный синтез на нульмерных локально компактных абелевых группах

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Abstract. Let G be a zero-dimensional locally compact Abelian group whose elements are compact, $C(G)$ the space of continuous complex-valued functions on the group G. A closed linear subspace $\mathcal{H} \subseteq C(G)$ is called invariant subspace, if it is invariant with respect to translations $\tau_y : f(x) \mapsto f(x+y)$, $y \in G$. We prove that any invariant subspace $\mathcal{H}$ admits spectral synthesis, which means that $\mathcal{H}$ coincides with the closure of the linear span of all characters of the group G contained in $\mathcal{H}$.

Keywords: zero–dimensional groups; characters; harmonic analysis; spectral synthesis; invariant subspaces

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Аннотация. Пусть G — нульмерная локально компактная абелева группа, все элементы которой компактны, $C(G)$ — пространство всех непрерывных комплекснозначных функций на группе G. Замкнутое линейное подпространство $\mathcal{H} \subseteq C(G)$ называется инвариантным подпространством, если оно инвариантно относительно сдвигов $\tau_y : f(x) \mapsto f(x+y)$, $y \in G$. В работе доказывается, что любое инвариантное подпространство $\mathcal{H}$ допускает спектральный синтез, то есть $\mathcal{H}$ совпадает с замыканием линейной оболочки всех содержащихся в $\mathcal{H}$ характеров группы G.

Ключевые слова: нульмерные группы; характеры; гармонический анализ; спектральный синтез; инвариантные подпространства

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1. General definitions

Let G be a locally compact Abelian group (LCA–group), $\mathcal{F}$ be a locally convex topological vector space that consists of complex-valued functions on the group G. This space is called a translation invariant space if it is invariant under translations (shifts)

$$\tau_y : f(x) \mapsto f(x+y), \quad f \in \mathcal{F},\ y \in G,$$

and all operators τ_y on the space $\mathcal{F}$ are continuous. A closed linear subspace $\mathcal{H} \subseteq \mathcal{F}$ is called an invariant subspace if $\tau_y(\mathcal{H}) \subseteq \mathcal{H}$ for any $y \in G$.

A continuous homomorphism of G into the multiplicative group $\mathbb{C}_* = \mathbb{C} \setminus \{0\}$ of nonzero complex numbers is called an *exponential functions* or *generalized character* on G. A continuous homomorphism of G into the group $\mathbb{T} := \{z \in \mathbb{C} : |z| = 1\}$ is called a character of G.

Continuous homomorphisms of G into the additive group of comlex numbers are called *additive* functions. A function $x \mapsto P(a_1(x), \ldots, a_m(x))$ on G is called a *polynomial* if P is a complex polynomial in m variables and $a_1, \ldots, a_m$ are additive functions. A product of a polynomial and an exponential function is called an *exponential monomial*, and linear combinations of of exponential monomials are called *exponential polynomials*.

Let $\mathcal{F}$ be a translation invariant space on G and $\mathcal{H}$ be an invariant subspace in $\mathcal{F}$.

D e f i n i t i o n 1.1. An invariant subspace $\mathcal{H}$ *admits spectral synthesis* if $\mathcal{H}$ coincides with the closed linear span in $\mathcal{F}$ of all exponential monomials that belong to $\mathcal{H}$. We say that a translation invariant space $\mathcal{F}$ has the spectral synthesis property if any invariant subspace $\mathcal{H} \subseteq \mathcal{F}$ admits spectral synthesis.

2. Examples of spectral synthesis

In this section we give some examples of spectral synthesis.

1. $G = (\mathbb{R}, +)$

Any exponential monomial on $\mathbb{R}$ has the form $f(x) = P(x)\, e^{\lambda x}$, where $x \in \mathbb{R}$, $\lambda \in \mathbb{C}$, $P(x)$ is a polynomial. The function spaces $C(\mathbb{R})$ of all continuous functions and $\mathcal{E}(\mathbb{R}) = C^\infty(\mathbb{R})$ of all infinitely differentiable functions (all classical function spaces are equipped with their usual topologies) have the spectral synthesis property. This is result of L. Schwartz [1]. Some other examples of functions spaces on $\mathbb{R}$ with spectral synthesis property were studied in the papers of J. E. Gilbert [2] and S. S. Platonov [3].

2. $G = (\mathbb{R}^n, +)$, $n \geq 2$

Any exponential monomial on $\mathbb{R}^n$ has the form $f(x) = P(x)\, e^{\lambda x}$, where $x = (x_1, \ldots, x_n) \in \mathbb{R}^n$, $\lambda = (\lambda_1, \ldots, \lambda_n) \in \mathbb{C}^n$, $\lambda x = \lambda_1 x_1 + \cdots + \lambda_n x_n$, $P(x)$ is a polynomial in x. In [1] L. Schwartz conjectured that the spaces $C(\mathbb{R}^n)$ and $\mathcal{E}(\mathbb{R}^n) = C^\infty(\mathbb{R}^n)$ have the spectral synthesis property. This conjecture turned out to be false. In 1975, D. I. Gurevich [4] costructed an example of an invariant subspace $\mathcal{H} \subset \mathcal{E}(\mathbb{R}^2)$ containing no exponential monomials. Nevertheless, L. Schwartz [5] proved that the space $\mathcal{S}'(\mathbb{R}^n)$ of all tempered distributions on $\mathbb{R}^n$ has the spectral synthesis property.

3. G is a discrete group

For the case when G is a discrete group, the most natural function space is the space $C(G)$ consisting of all complex-valued functions on G with the topology of pointwise convergence. The case $G = \mathbb{Z}^n$ was studied by M. Lefranc [6]. He proved that the space $C(\mathbb{Z}^n)$ has the spectral synthesis property. Some results about the spectral synthesis on the discrete groups were considered in [7]. In particular, the space $C(G)$ has the spectral synthesis property if G is a finitely generated Abelian group [8] or a torsion Abelian group [9]. In [10] M. Laczkovich and L. Székelyhidi proved that the spectral synthesis in the space $C(G)$ holds on a discrete Abelian group G if and only if the torsion free rank of G is finite. For the case when G is a finitely generated discrete Abelian group and $\mathcal{F}$ is the space of all exponential growth functions on G the spectral synthesis property was proved in [11].

3. Main results

Let G be a LCA-group. An element $x \in G$ is called a compact element if the smallest closed subgroup of G, which contains x, is compact.

Let G be a LCA-group, such that all elements of G are compact. Any generalized character of G is a usual character and any additive function on G is zero. Any exponential monomial on G has the form $\lambda\chi(x)$, where $\lambda \in \mathbb{C}$, $\chi(x)$ is a character of G.

P r o p o s i t i o n 3.1. *Let $\mathcal{F}$ be a translation invariant space on G, $\mathcal{H}$ be an invariant subspace in $\mathcal{F}$. If G is a LCA-group, such that all elements of G are compact, then $\mathcal{H}$ admits spectral synthesis if and only if $\mathcal{H}$ coicides with the closed linear span in $\mathcal{F}$ of all characters of G that belong to $\mathcal{H}$.*

For any LCA-group G let $\widehat{G}$ be the set of all characters of G. The set $\widehat{G}$ is a LCA-group (dual group of G) with compact-open topology and multiplication being defined as the pointwise multiplication of functions.

For any invariant subspace $\mathcal{H} \subseteq \mathcal{F}$, the set $\sigma(\mathcal{H}) := \{\chi \in \widehat{G} : \chi \in \mathcal{H}\}$. is called the *spectrum* of $\mathcal{H}$.

If G is a LCA-group, such that all elements of G are compact, and invariant subspace $\mathcal{H}$ admits spectral synthesis, then $\mathcal{H}$ can be recovered uniquely by its spectrum $\sigma(\mathcal{H})$.

A locally compact topological space X is called zero-dimensional if compact open subsets of X form a basis of topology. A locally compact Hausdorff topological space X is zero-dimensional if and only if X is totally disconnected, that is any subset of X, which contains more then one point, is disconnected.

Theorem 3.1. *Let G be a locally compact zero-dimensional Abelian group, such that all elements of G are compact. Then:* 1) *the space $C(G)$ of all continuous functions on G has the spectral synthesis property;* 2) *a subset $\sigma \subseteq \widehat{G}$ is the spectrum of some invariant subspace of $C(G)$ if and only if σ is closed subset of $\widehat{G}$.*

4. Some examples of zero-dimensional LCA-groups, all elements of which are compact

1. Let $\{n_k\}_{k\in\mathbb{Z}}$ be a two-side sequence, $n_k \in \mathbb{N}$, $n_k \geqslant 2$. Let

$$\widetilde{G} = \bigoplus_{k\in\mathbb{Z}} \mathbb{Z}_{n_k},$$

where $\mathbb{Z}_n$ is the cyclic group of order n. Every $\mathbb{Z}_{n_k}$ is a discrete group and $\widetilde{G}$ is a compact group. Any element of $\widetilde{G}$ has the form

$$x = \{x_k\}_{k\in\mathbb{Z}}, \qquad x_k \in \mathbb{Z}_{n_k}.$$

Let G be a subgroup of $\widetilde{G}$ that consist of all elements

$$x = \{x_k\} \in \widetilde{G} : \exists N(x) \in \mathbb{Z} \quad \forall k < N(x) \quad x_k = 0.$$

The group G is locally compact, zero-dimensional and all elements of G are compact.

If $n_k = 2 \quad \forall k \in \mathbb{Z}$, then we have the locally compact Cantor dyadic group. The harmonic analysis on this group closely connected with Fourier–Walch harmonic analysis (see [12]).

2. Let $\mathbb{Q}_p$ be the group of p-adic numbers. Any element $x \in \mathbb{Q}_p$ can be identified with a formal series

$$x = \sum_{k\geqslant N(x)} x_k p^k, \quad x_k \in \{0, 1, \dots, p-1\}, \quad N(x) \in \mathbb{Z}.$$

The group $\mathbb{Q}_p$ is locally compact, zero-dimensional and all elements of G are compact.

Also, for any two-side sequence $\mathbf{a} = (a_k)_{k\in\mathbb{Z}}$, $a_k \in \mathbb{N}$, $a_k \geqslant 2$, there exist the group $\mathbb{Q}_{\mathbf{a}}$ of generalized $\mathbf{a}$-adic numbers (see [13]). The group $\mathbb{Q}_{\mathbf{a}}$ is locally compact, zero-dimensional and all elements of G are compact. A zero-dimensional LCA-group G with countable base of topology, such that all elements of G are compact, is called a Vilenkin group. Harmonic analysis on such groups was studied in [14].

5. On the ideal structure of algebras of locally constant functions

Let X be a zero-dimensional Hausdorff locally compact topological space. Let $\tau_{co}(X)$ be the set of all compact open subsets of X. The set $\tau_{co}(X)$ forms a basis of topology of X. Any finite set $\alpha = \{U_1, \dots, U_n\}$ of mutually disjoint subsets $U_i \in \tau_{co}(X)$ is called a discrete system of subsets of X. Let $\mathfrak{M}(X)$ be the set of all discrete systems of subsets of X. For $\alpha = \{U_1, \dots, U_n\} \in \mathfrak{M}(X)$, the support of α is the set

$$\operatorname{supp} \alpha := \bigcup_{i=1}^{n} U_i.$$

A function f on X is called locally constant if for any $x \in X$ there exist neighbourhood $U = U(x)$ of x on which f is constant. Denote by $\mathcal{D}(X)$ the set of all locally constant complex-valued functions on X with compact support. The set $\mathcal{D}(X)$ is a linear space. Now we define a topology on $\mathcal{D}(X)$.

For any $\alpha \in \{U_1, \dots, U_n\} \in \mathfrak{M}(X)$ let $\mathcal{D}_\alpha(X)$ be the set of functions of the form $f = \sum_{i=1}^{n} c_i I_{U_i}$, where $c_i \in \mathbb{C}$, I_U is the characteristic function of U. The set $\mathcal{D}_\alpha(X)$ is n-dimensional vector space. With respect to the uniform norm

$$\|f\|_\infty := \sup_{x\in X} |f(x)|$$

the set $\mathcal{D}_\alpha(X)$ is a Banach space. We equip the space

$$\mathcal{D}(X) = \bigcup_{\alpha\in\mathfrak{M}(X)} \mathcal{D}_\alpha(X)$$

with the topology of inductive limits of the Banach spaces $\mathcal{D}_\alpha(X)$, that is a topology of $\mathcal{D}(X)$ is the weakest locally convex topology for which all inclusions $\mathcal{D}_\alpha(X) \subseteq \mathcal{D}(X)$ are continuous. Then $\mathcal{D}(X)$ is locally convex space. With respect to the pointwise multiplication of functions, $\mathcal{D}(X)$ is a topological algebra.

Let $\mathcal{I}$ be an ideal of the algebra $\mathcal{D}(X)$. Denote by $N(\mathcal{I})$ the set of zeros of all functions from $\mathcal{I}$, that is

$$N(\mathcal{I}) := \{x \in X : f(x) = 0 \quad \forall x \in \mathcal{I}\}.$$

The set $N(\mathcal{I})$ is called zero set of $\mathcal{I}$.

For any closed subset $A \subseteq X$ denote by $\mathcal{I}_A$ the set of all functions $f \in \mathcal{D}(X)$, such that $f(x) = 0$ for any $x \in A$. The set $\mathcal{I}_A$ is a closed ideal of $\mathcal{D}(X)$.

Theorem 5.1. *Let $\mathcal{I}$ be an ideal of the algebra $\mathcal{D}(X)$ then $\mathcal{I}_{N(\mathcal{I})} = \mathcal{I}$.*

Corollary 5.1. *Any ideal of the topological algebra $\mathcal{D}(X)$ is closed.*

6. The proof of Theorem 3.1

Let G be a zero-dimensional LCA-group, $C(G)$ be the set of all continuous functions on G, $\mathcal{D}(G)$ be the set of locally constant functions with compact support on G. By $\mathcal{M}_c(G)$ we denote the set of complex-valued Radon measures with compact support on G. The space $\mathcal{M}_c(G)$ can be identified with the dual space of $C(G)$ with respect to the duality

$$\langle \mu, f \rangle := \int\limits_G f(x)\, d\mu(x), \quad f \in C(G), \quad \mu \in \mathcal{M}_c(G).$$

The space $\mathcal{M}_c(G)$ is a locally convex space with respect to the weak topology $\sigma(\mathcal{M}_c(G), C(G))$.

Let $\mu_1, \mu_2 \in \mathcal{M}_c(G)$. A convolution $\mu_1 * \mu_2$ is defined by formula

$$\langle \mu_1 * \mu_2, \varphi \rangle := \int\limits_G \int\limits_G \varphi(x+y)\, d\mu_1(x)\, d\mu_2(y),$$

where $\varphi \in C(G)$.

The set $\mathcal{M}_c(G)$ is a commutative topological algebra with convolution as multiplication. For any closed linear subspace $\mathcal{H} \subseteq C(G)$, let $\mathcal{H}^\perp$ be its annihilator in $\mathcal{M}_c(G)$ that is

$$\mathcal{H}^\perp := \{\mu \in \mathcal{M}_c(G) : \langle \mu, f \rangle = 0 \quad \forall f \in \mathcal{H}\}.$$

The mapping $\mathcal{H} \mapsto \mathcal{H}^\perp$ is one-to-one correspondence between the set of all invariant subspaces of $C(G)$ and the set of all closed ideals of topological algebra $\mathcal{M}_c(G)$.

Let $\mathcal{D}(G)$ be the set of all locally constant complex-valued functions on G with compact support. The set $\mathcal{D}(G)$ is a commutative topological algebra with convolution as multiplication:

$$(f_1 * f_2)(x) = \int\limits_G f_1(x-y) f_2(y) dy. \quad f_1,\ f_2 \in \mathcal{D}(G).$$

We will denote this topological algebra by $\mathcal{D}_{conv}(G)$.

For any topological algebra $\mathcal{A}$ we will denote by $s(\mathcal{A})$ the set of all closed ideals of $\mathcal{A}$. In particular we have the sets $s(\mathcal{M}_c(G))$ and $s(\mathcal{D}_{conv}(G))$. Using identification a function $f \in \mathcal{D}(G)$ with the measure $f(x)\,dx$, we have inclusion $\mathcal{D}(G) \subseteq \mathcal{M}_c(G)$. The maps

$$\rho : s(\mathcal{M}_c(G)) \mapsto s(\mathcal{D}_{conv}(G)) \text{ and } \tilde{\rho} : s(\mathcal{D}_{conv}(G)) \mapsto s(\mathcal{M}_c(G))$$

are defined by formulas:

$$\rho(\mathcal{H}) := \mathcal{H} \cap \mathcal{D}(G), \quad \mathcal{H} \in s(\mathcal{M}_c(G)), \quad \tilde{\rho}(\mathcal{H}_0) := [\mathcal{H}_0], \qquad \mathcal{H}_0 \in s(\mathcal{D}_{conv}(G)),$$

where $[\mathcal{H}_0]$ is the closure of $\mathcal{H}_0$ in the space $\mathcal{M}_c(G)$.

P r o p o s i t i o n 6.1. *The mapping ρ is a biection of set $s(\mathcal{M}_c(G))$ onto the set $s(\mathcal{D}_{conv}(G))$. The inverse mapping ρ^{-1} coincide with $\tilde{\rho}$.*

Let G be a LCA-group and $\widehat{G}$ be the dual group. It can be proved that LCA-group G is zero-dimensional group, all elements of which are compact, if and only if the dual group $\widehat{G}$ is zero-dimensional group, all elements of which are compact.

The Fourier transform of a function $f \in L_1(G)$ is the function $\widehat{f}$ on the dual group $\widehat{G}$ which is defined by formula

$$\widehat{f}(\chi) := \int\limits_G f(x)\,\overline{\chi(x)}\,dx, \quad \chi \in \widehat{G}.$$

In particular, the Fourier transform is defined for any function $f \in \mathcal{D}(G)$. The mapping $\Phi : f \mapsto \widehat{f}$ is also called the Fourier transform.

P r o p o s i t i o n 6.2. *If G is a is zero-dimensional group, all elements of which are compact, then the Fourier transform Φ is an isomorphism of the topological vector space $\mathcal{D}(G)$ into the topological vector space $\mathcal{D}(\widehat{G})$.*

Corollary 6.1. *The mapping Φ is an isomorphism of topological algebra $\mathcal{D}_{conv}(G)$ into the topological algebra $\mathcal{D}_{mult}(\widehat{G})$.*

P r o o f of **Theorem 3.1** Let $\mathcal{H}$ be an invariant subspace of $C(G)$, $\mathcal{H}^\perp$ be its annihilator in $\mathcal{M}_c(G)$, $\mathcal{I} = \mathcal{H}^\perp \cap \mathcal{D}(G)$, $\widehat{\mathcal{I}} = \Phi(\mathcal{I})$. Then $\mathcal{I}$ is a closed ideal of $\mathcal{D}_{conv}(G)$, and $\widehat{\mathcal{I}}$ is a closed ideal of $\mathcal{D}_{mult}(\widehat{G})$. We will say that the ideal $\widehat{\mathcal{I}}$ corresponds to the invariant subspace $\mathcal{H}$.

Let $\chi \in \widehat{G}$. One can prove that $\chi \in \mathcal{H}$ if and only if the point χ belongs to zero set of the ideal $\widehat{\mathcal{I}}$. Thus the spectrum $\sigma(\mathcal{H})$ of invariant subspace $\mathcal{H}$ is the same as zero set $N(\widehat{\mathcal{I}})$ of corresponding ideal $\widehat{\mathcal{I}} \subseteq \mathcal{D}_{mult}(\widehat{G})$.

Let $\mathcal{H}$ be an invariant subspace of $C(G)$. Denote by $\mathcal{H}_1$ a closed linear subspace of $C(G)$, that coincides with the closed linear span in C(G) of all characters of G that belong to $\mathcal{H}$. Then $\mathcal{H}_1$ is also an invariant subspace of $C(G)$ and $\sigma(\mathcal{H}) = \sigma(\mathcal{H}_1)$. Let $\mathcal{I}_1 = \mathcal{H}_1^\perp \cap \mathcal{D}(G)$, $\widehat{\mathcal{I}}_1 = \Phi(\mathcal{I}_1)$. Since $N(\widehat{\mathcal{I}}) = N(\widehat{\mathcal{I}}_1)$ then we have $\widehat{\mathcal{I}} = \widehat{\mathcal{I}}_1$ by Theorem 5.1, and from Proposition 6.1 we have $\mathcal{H} = \mathcal{H}_1$. This completes the proof of Theorem 3.1.

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